# The Paradoxes, Perplexities, and Power of Factor Analysis

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# Plan of Presentation

1) Factor Analysis
2) Problems with Factor Analysis Interpretation
   i. Tests to Reject Structural Factor Models
   ii. Causal Relations between Factors
   iii. Factor Analysis with Different Distributional Assessments
3) Provocative Example
4) Reinterpretation of Factor Analysis


- ❖ VanderWeele, T.J. and Batty, C.J.K. (2023). On the dimensional indeterminacy of one-wave factor analysis under causal effects. Journal of Causal Inference (https://doi.org/10.1515/jci-2022-0074).
- ❖ VanderWeele, T.J. and Vansteelandt, S. (2022). A statistical test to reject the structural interpretation of a latent factor model. Journal of the Royal Statistical Society, Series B, 84:2032-2054.
- ❖ VanderWeele, T.J. (2022). Constructed measures and causal inference: towards a new model of measurement for psychosocial constructs. Epidemiology, 33:141-151.

# Factor Analysis

For a set of observed indicators **X**=($X_1$,…, $X_d$), potentially used to assess some psychosocial construct, statistical factor analysis is often employed to assess the dimensionality of ($X_1$,…, $X_d$)

We attempt to evaluate the extent to which a set of latent factors $\eta$ can explain the variability in **X**=($X_1$,…, $X_d$) such that:

$$\mathbf{X} = \Lambda\eta + \boldsymbol{\epsilon}$$

Often it is of interest to assess whether a univariate latent variable $\eta$ is sufficient

When it is, then sometimes the mean of ($X_1$,…, $X_d$) is used as an assessment of the construct

When it is not sufficient, then factor analysis is sometimes used to try to group the indicators so as to correspond to distinct factors

# Dimensionality Assessment in Factor Analysis

Evidence for the number of factors often involves examining:

(i) The $\boldsymbol{\chi}^2$ test for comparing the covariance of $\Lambda\eta + \boldsymbol{\epsilon}$ to an unconstrainted covariance for **X**

(ii) Other goodness of fit statistics less dependent upon sample size

(iii) Eigenvalues of Cov(**X**)

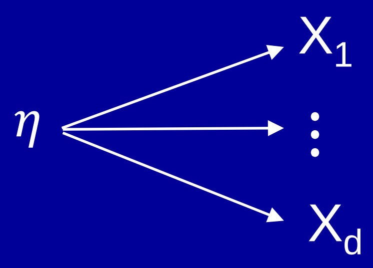
(iv) The magnitude of factor loadings in $\Lambda$

(v) Various other rules of thumb

There is no universal set of practices

# Factor Analysis Interpretation

If there is seems to be evidence for unidimensionality of the shared variance of indicators **X**=($X_1$,…, $X_d$) then it is frequently assumed that there is some underlying unidimensional continuous latent variable $\eta$ that gives rise to the indicators




so that $X_i = \lambda_i \eta + \varepsilon_i$

Often it is implicitly assumed that the factor in some sense "exists" and has a meaningful scientific interpretation as a continuous variable and is causally efficacious for outcomes

When more than one factor is needed, then it is often assumed that multiple such continuous latent variables exist

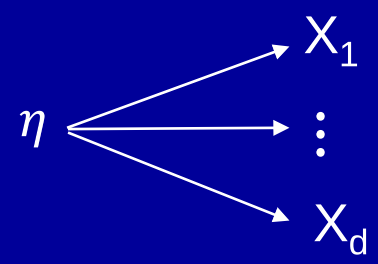
# Challenging Factor Analysis Interpretation

In this talk I would like to:

- Challenge the naïve factor analysis interpretation
- Challenge the notion that there are underlying causally efficacious univariate latent variables corresponding to factors
- Challenge the notion that we can identify the dimensionality of causally efficacious “factors” with one wave of data
- Challenge dimensionality assessments when both positively and negatively worded items are used
- Offer a series of empirical examples illustrating these issues
- Offer a reinterpretation of factor analysis acknowledging that in spite of these challenges it can still be a very powerful technique

# Problem 1. Structural Interpretation

If there seems to be evidence for unidimensionality of the shared variance of indicators $\mathbf{X}=(X_1,\ldots, X_d)$ (often using techniques of factor analysis) then it is frequently assumed that some underlying unidimensional continuous latent variable $\eta$ that gives rise to the indicators



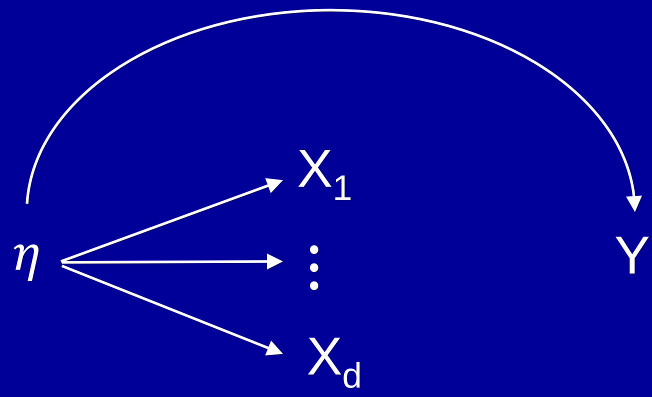

Often it is assumed that the latent variable $\eta$ exists and is causally efficacious for various outcomes and the indicators $(X_1,\ldots, X_d)$ are just imprecise assessments of $\eta$

# Structural vs. Statistical Factors

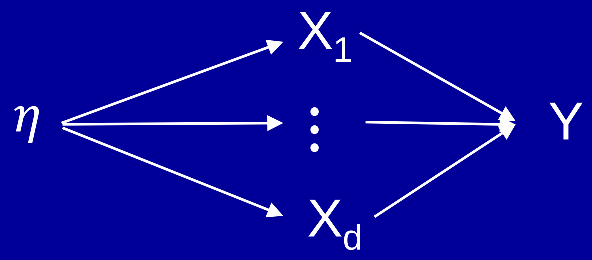
However the basic statistical factor model

$$X_i = \lambda_i \eta + \varepsilon_i \qquad (1)$$

is consistent with η being causally efficacious:




But it is also alternatively entirely consistent with the indicators being causally efficacious:



# Structural Factors

Structural Factors: We will say that a factor model

$$X_i = \lambda_i \eta + \varepsilon_i$$

is structural if the indicators, $(X_1, \ldots, X_d)$, do not have causal effects on anything subsequent, and if they are themselves only affected by antecedents through the latent variable $\eta$.

Causal Diagrams: On a causal diagram, a factor $\eta$ would be structural if there are no arrows going out of $(X_1, \ldots, X_d)$ and no arrows going into $(X_1, \ldots, X_d)$ except from $\eta$.

Independence: On a causal diagram this also implies for other variables Z on the diagram, Z will be independent of $(X_1, \ldots, X_d)$ conditional on $\eta$.

This is what is assumed in most SEMs with latent variables (Bollen, 1989)

# Empirical Implications

The assumption that a factor is structural is so strong that it has empirically testable implications even though the latent factor $\eta$ is never observed

**Theorem 1.** *Suppose that Z is independent of* $(X_1, \ldots, X_d)$ *conditional on* $\eta$ *and that the basic latent factor model in Equation (1) holds, then for any i and j, and any values z and* $z^*$*, we must have* $\lambda_i \left\{E(X_j|Z = z) - E\left(X_j|Z = z^*\right)\right\} = \lambda_j \left\{E(X_i|Z = z) - E(X_i|Z = z^*)\right\}$.

Corollary: For a randomized treatment T, a structural factor implies:

$$\{E[X_j|T = 1] - E[X_j|T = 0]\}/\lambda_j = \{E[X_i|T = 1] - \{E[X_i|T = 0]\}/\lambda_i.$$

Corollary: For any outcome Y, a structural factor implies:

$$\{E(X_j|Y = 1) - E(X_j|Y = 0)\}/\lambda_j = \{E(X_i|Y = 1) - \{E(X_i|Y = 0)\}/\lambda_i$$

# Statistical Test for Structural Latents (VanderWeele and Vansteelandt, 2022)

We can use these empirical implications to develop a statistical test to evaluate the null of a structural latent factor model if we…

define $\gamma_i = E(X_i|Z=1)$ and $\beta_w = \{E(X_1|Z=w) - E(X_1|Z=1)\}$ then under the null hypothesis we can parameterize $E(X_i|Z=z)$ as:

$$E(X_i|Z=z) = \gamma_i + \frac{\lambda_i}{\lambda_1}\sum_{w=2}^{p} \beta_w I(z=w) \tag{2}$$

for $i=1,\ldots,d$, where $\gamma_i, \mathrm{i} = 1, \ldots, \mathrm{d}$ and $\beta_w, w = 2, \ldots, p$ are unknown. Let $U_k$ be a $(p \times d)$-dimensional vector with elements $I(Z_k = z)\left\{X_{ik} - \gamma_i - \frac{\lambda_i}{\lambda_1}\sum_{w=2}^{p} \beta_w I(z=w)\right\}$ for $z=1,\ldots,p$.

We can construct a generalized methods of moments estimator (Newey and McFadden, 1994) under the null by minimizing

$$T_0 = N\left(\frac{1}{N}\sum_{k=1}^{N} U_k^T\right)\Sigma^{-1}\left(\frac{1}{N}\sum_{k=1}^{N} U_k\right),$$

With respect to $\boldsymbol{\gamma}_i$ and $\boldsymbol{\beta}_w$ where $\boldsymbol{\Sigma}$ is the empirical covariance matrix of $U_k$, or a modification if $\boldsymbol{\lambda}_i$ are estimated (as is usually the case)

The minima will follow a $\chi^2$ with (d-1) x (p-1) degrees of freedom

We can also construct alternative tests without estimating $\boldsymbol{\lambda}_i$, and relying on weaker distributional assumptions, if Z has more than 2 levels

# Application 1: Satisfaction with Life Scale

One of the most widely used subjective well-being scales is Diener et al.'s (1985) Satisfaction with Life Scale (>40,000 citations)

| Item number | Item content[a] |
|---|---|
| 1 | In most ways my life is close to my ideal |
| 2 | The conditions in my life are excellent |
| 3 | I am satisfied with my life |
| 4 | So far I have gotten the important things I want in life |
| 5 | If I could live my life over, I would change almost nothing |

[a]Individuals are asked to rate their level of agreement to each item from "strongly disagree" (1) to "strongly agree" (7)

- Good psychometric properties: Cronbach's alpha is high and a single factor seems to explain a considerable proportion of the variance across item responses (Diener et al., 1985; Pavot and Diener, 1993).

# Application 1: Satisfaction with Life Scale

Kim et al. (2021) examine associations with all-cause mortality with Health and Retirement Study (HRS) Data (N=12,998, mean age = 66):

- Examined associations of tertiles of life satisfaction in 2010/2012 with 4-year mortality
- Controlled for sociodemographic characteristics (age, sex, race/ethnicity, marital status, annual household income, total wealth, level of education, employment status, health insurance, geographic region), childhood abuse, religious service attendance, health conditions and behaviors (diabetes, hypertension, stroke, cancer, heart disease, lung disease, arthritis, overweight/obesity, chronic pain, binge drinking, current smoking status, physical activity, sleep problems), various other aspects of psychological well-being (positive affect, optimism, purpose in life, mastery, depressive symptoms, hopelessness, negative affect, loneliness, social integration), and personality factors (openness, conscientiousness, extraversion, agreeableness, neuroticism).

Those in the top tertile of life-satisfaction were 0.74 (95%: 0.64, 0.87) times less likely to die during the four years of follow-up than those in the bottom tertile

# Application 1: Satisfaction with Life Scale

Supplementary analyses examined associations by indicator:

| | |
|---|---|
| "In most ways my life is close to my ideal" | (RR=0.75; 95% CI: 0·61, 0·91) |
| "The conditions of my life are excellent" | (RR=0.79; 95% CI: 0·66, 0·95) |
| "I am satisfied with my life" | (RR=0.72; 95% CI: 0·62, 0·84) |
| "So far I have gotten the important things I want in life" | (RR=0.85; 95% CI: 0·73, 0·99) |
| "If I could live my life over, I would change almost nothing" | (RR=0.98; 95% CI: 0·83, 1·16) |

Applying the likelihood ratio test with Z= 4-year mortality:
$X^2$=57.25 with df=(5-1)(2-1)=4; strong evidence against the null ($p=1.1\times10^{-11}$)

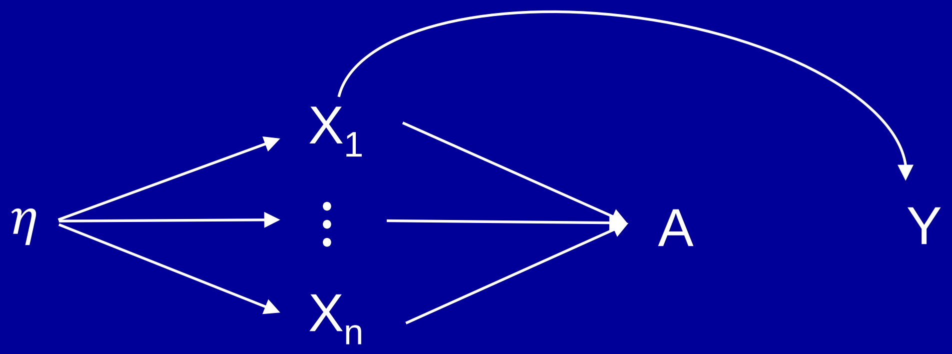
There is no underlying continuous univariate "life satisfaction" latent $\boldsymbol{\eta}$ to which these indicators correspond with uniform effects on mortality:

- This does not mean the "Satisfaction with Life Scale" is bad
- It may be a perfectly reasonable summary outcome
- But there is no underlying univariate "latent construct"

# Implications

Factor analytic models can completely obscure relevant causal distinctions

It may be that only a single indicator is causally relevant for the outcome even if a single factor seems to statistically fit the data well




Implications:

Evidence for a single structural factor needs to be established not presumed

Without such evidence, indicator-by-indicator analyses may be preferable

Caution: Most psycho-social constructs are likely inherently multi-dimensional

❖ VanderWeele, T.J. (2022). Constructed measures and causal inference: towards a new model of measurement for psychosocial constructs. Epidemiology, 33:141-151.

# Why do we assume a unidimensional construct?

- One factor often statistically fits the data well
- It seems like a natural leap
- It makes analysis easier
- It is often standard practice
- We don’t examine the relevant evidence for a “structural” factor

Unfortunately, this is not the only issue with factor analysis interpretation

What happens if the factors causally affect one another…?

E.g. Mental health: anxiety causes depression, and vice versa

We might then confuse causal and conceptual relationships…

# Problem 2. Causal Relations Between Factors

At time t consider p items: $Y^t = (Y_1^t, \dots, Y_p^t)$

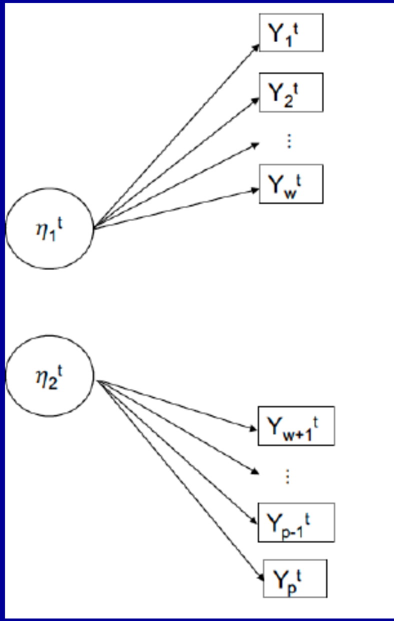
And m latent factors: $\eta^t = (\eta_1^t, \dots, \eta_m^t)$

The standard factor analytic model with independent errors is given by:

$$Y^t = \Lambda\eta^t + \varepsilon$$

where $\Lambda$ is an $p \times m$ matrix and $\varepsilon$ is an $p \times 1$ vector of independent normally distributed random variables

Example: Two factors with distinct items…

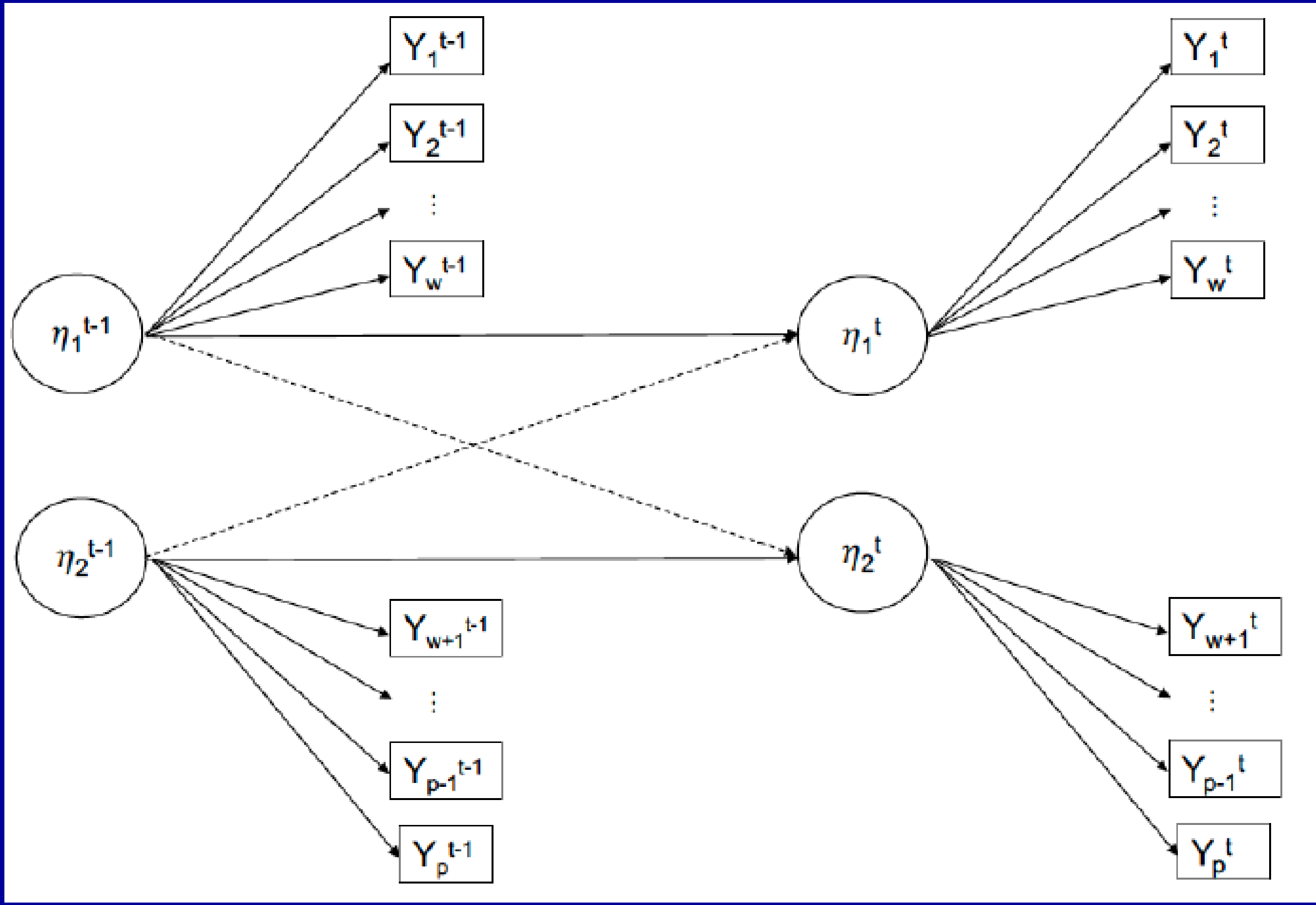


# Causal Relations between Factors

But what if the factors causally affect one another over time?

$$\eta^t = B\eta^{t-1} + W^t$$

where $B$ is an $m \times m$ matrix and $W^t$ is a $m \times 1$ vector of random errors



# Dynamic Factor Analysis in Equilibrium (VanderWeele and Batty, 2023)

**Theorem 1** *Suppose indicators* $Y^t = (Y_1^t, \ldots, Y_p^t)'$ *at time t are related to a set of m latent factors* $\eta^t = (\eta_1^t, \ldots, \eta_m^t)'$ *by:*

$$Y^t = \Lambda\eta^t + \varepsilon$$

*and that* $\eta^t = B\eta^{t-1} + W^t$ *where the variables* $W^t$ *are independently normally distributed with potentially distinct parameters at each t. Let* $B = QDQ^{-1}$ *be Jordan decomposition of B where* **D** *is an* $m\times m$ *matrix of Jordan normal form. Suppose it is the case that (i) as* $t \to \infty$, $Y^t$ *converges in distribution to some random variable* $Y^*$, *(ii) B is invertible and (iii) the random variables* $W^t$ *decay sufficiently quickly such that as* $t \to \infty$, $(\sum_{k=1}^{t} B^{-k} W^k)$ *converges in distribution to some normally distributed variable* $W^*$, *then* $Y^*$ *will follow a factor model:*

$$Y^* = \Lambda^*\eta^* + \varepsilon$$

*with* $dim(\eta^*)= rank(D^*)$ *where* $D^* = \lim_{t\to\infty} D^t$.

**Corollary** *Under the conditions of Theorem 1 if* $dim(\eta^t)$=2 *then as* $t \to \infty$ *the resulting factor model:*

$$Y^* = \Lambda^*\eta^* + \varepsilon$$

*is such that* $dim(\eta^*)= 2$ *if and only if* $B=I$.

# Dynamic Factor Analysis in Equilibrium

Interpretation: Suppose that, in a given wave, conditional on the past, the items load completely independently on 2 separate factors, but at least one of these factors causally affects the other; as the causal process plays out over time, in equilibrium, in an exploratory factor analysis with 1 wave of data, a single factor will often be sufficient

Simulations indicate that in a relatively small number of steps, it may not be possible to distinguish the two factors

Implication: If there is any possibility that there is more than one factor with factors affecting each other, and a one-wave exploratory factor analysis indicates a single factor, then we essentially learn nothing

The goal of many psychometric measure efforts is often to establish that there is a single factor and that the indicators make up a reasonable scale, but it is not clear whether current practices are sufficient for this

# Application 2: Depression and Anxiety

Current 1-wave exploratory factor analysis practices are problematic and can potentially give rise to misleading conclusions

Factor analyses with anxiety and depression items suggest only 1 factor

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1993, Vol. 61, No. 4, 631–638


0022-006X/93/$3.00

## Distinguishing Depression and Anxiety in Self-Report: Evidence From Confirmatory Factor Analysis on Nonclinical and Clinical Samples

Lisa A. Feldman

Psychologists believe that anxiety and depression self-report scales tap distinct constructs. This assumption was tested by using confirmatory factor analysis on mood data from nonclinical samples (K. S. Dobson, 1985a; I. H. Gotlib, 1984; J. Tanaka-Matsumi & V. A. Kameoka, 1986) and a clinical sample (J. Mendels, N. Weinstein, & C. Cochrane, 1972). These analyses provide evidence that anxiety and depression self-report scales do not measure discriminant mood constructs and may therefore be better thought of as measures of general negative mood rather than as measures of anxiety and depression per se.

# Application 2: Depression and Anxiety

Norton S, Cosco T, Doyle F, Done J, Sacker A. The Hospital Anxiety and Depression Scale: a meta confirmatory factor analysis. Journal of psychosomatic research. 2013 Jan 1;74(1):74-81.

*Objective:* To systematically evaluate the latent structure of the Hospital Anxiety and Depression Scale (HADS) through reanalysis of previous studies and meta confirmatory factor analysis (CFA).

*Method:* Data from 28 samples were obtained from published studies concerning the latent structure of the HADS. Ten models were considered, including eight previously identified models and two bifactor models. The fit of each model was assessed separately in each sample and by meta CFA. Meta CFA was conducted using all samples and using subgroups consisting of community samples, cardiovascular disease samples and samples from studies administering the English language version of the HADS.

*Conclusion:* A bifactor structure provides the most acceptable empirical explanation for the HADS correlation structure. Due to the presence of a strong general factor, the HADS does not provide good separation between symptoms of anxiety and depression. We recommend it is best used as a measure of general distress.

But there is considerable evidence that anxiety causally gives rise to depression, and depression to anxiety (cf. e.g. Jacobson and Newman, 2017 meta-analysis)

The factor analysis results are thus exactly what one would expect with two distinct factors in the presence of causal effects

Moreover if we have multiple related "factors"/facets that affect one another the same phenomena will arise, and it will seem there is one "factor"



In how many other cases might similar issues arise…?

# Causation and Factor Analysis

Conclusion: Associations can arise from causal or conceptual relations; we need to distinguish between causal and conceptual relations

Causal Fallacy: Correlations implies causal

Measurement Fallacy: Correlation never implies causation (it always indicates a conceptual relationship)

The "measurement fallacy" is perhaps as problematic as the "causal fallacy"

Implications:

- Evidence for univariate factor structure of covariance does not mean all indicators are assessing the same thing
- Factor analyses with one wave of data should themselves be interpreted as characterizing associations among indicators that may be present either due to conceptual relations or due to causal relations concerning the underlying construct phenomena

The prior challenges concerned cases in which multidimensional phenomena could be confused as being univariate phenomena



However, the reverse can also occur…

# Problem 3. Truncated Assessments

In some surveys both positively- and negatively- worded items are used

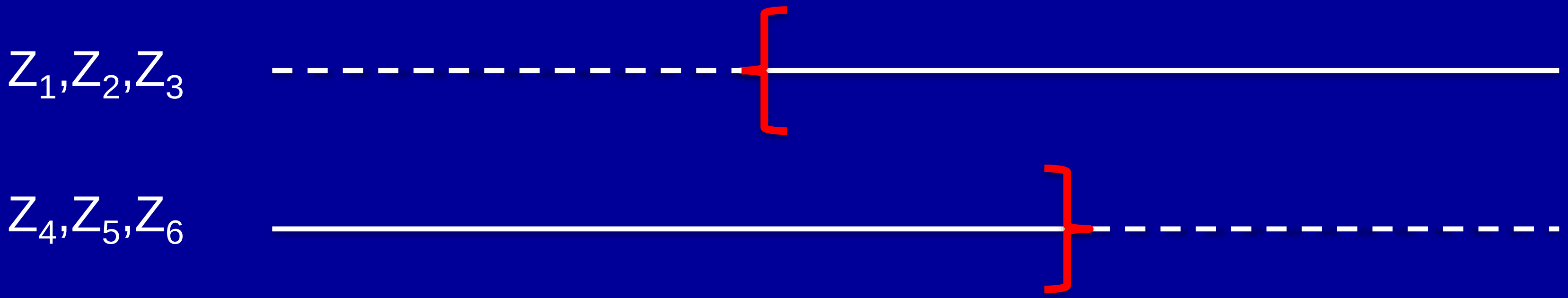
However, the indicators may be evaluating different parts of the distribution of the construct phenomena that is being assessed

Suppose we have six relatively reliable indicators $(Y_1, \ldots, Y_6)$ given by:

$$Y_i = (0.9)\eta + \varepsilon_i$$

where $\varepsilon_i$ is normally distributed with mean 0 and variance 0.19

Suppose further, however, that in the assessments, $(Z_1, \ldots, Z_6)$, that take place, the first three indicators are truncated below at -0.05, and the final three indicators are truncated from above at 0.05.




Here $Z_1, Z_2, Z_3$ provide more information when $\eta$ is positive, and $Z_4, Z_5, Z_6$ provide more information when $\eta$ is negative

# Truncated Assessments

Thus, if $Y_i = (0.9)\eta + \varepsilon_i$ with $\varepsilon_i$ ~N(0,0.19) then

Let $Z_i = \max(Y_i, -0.05)$ for *i*=1,2,3 and $Z_i = \min(Y_i, 0.05)$ for *i*=4,5,6.

Now let $X_i = \frac{\{Z_i - E(Z_i)\}}{SD(Z_i)}$

Once we use ($X_1$,…,$X_6$) it may no longer be intuitively clear that the first three indicators are simply assessing lower parts of the distribution of $\eta$ than are the second three indicators

What happens if we employ factor analysis…?

The analyses that follow use simulated data with a sample size of 1,000

# Factor Analysis with Truncated Assessments (VanderWeele et al. 2023)

- One Factor Model: Under the null hypothesis that one latent factor (with 6 factor loadings) is sufficient, as compared with an unconstrained model for the correlation matrix with 15 parameters, a likelihood ratio $\chi^2$ test-statistic with 9 degrees of freedom gives a p-value of $2.97 \times 10^{-306}$

- Two Factor Model: If we fit a factor model with two factors, the factor loadings using varimax rotation are $(0.90, 0.89, 0.88, 0.39, 0.39, 0.40)'$ for the first factor and $(0.12, 0.13, 0.14, 0.77, 0.82, 0.78)'$ for the second factor. Under the null hypothesis that two latent factors are sufficient, as compared with an unconstrained model for the correlation matrix, a likelihood ratio $\chi^2$ test-statistic with 4 degrees of freedom gives a p-value of 0.88, suggesting that the model with two factors is a reasonably good fit to explain the covariance of $(X_1, \ldots, X_6)$

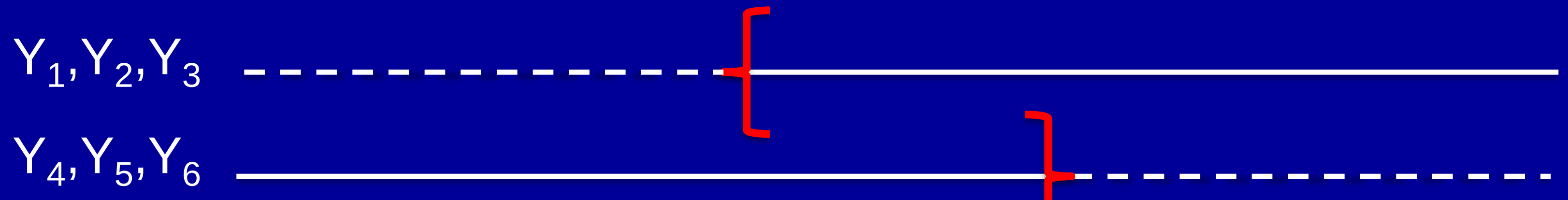
- We seem to have evidence for two factors, but… the actual underlying variable $\eta$, generating all of then data, is univariate

# Application 3: Optimism and Pessimism

- The Life Orientation Test-Revised of optimism uses 6 items (3 negatively worded, and 3 positively worded)

  (1) "I'm always optimistic about my future"
  (2) "In uncertain times, I usually expect the best"
  (3) "Overall, I expect more good things to happen to me than bad"
  (4) "If something can go wrong for me it will" (reverse coded)
  (5) "I hardly ever expect things to go my way" (reverse coded)
  (6) "I rarely count on good things happening to me" (reverse coded)

- Factor analyses consistently indicate a two-factor structure (Chiesi et al., 2013; Monzani et al., 2014)
- This is sometimes interpreted as optimism and pessimism being "independent constructs" (Herzberg et al., 2006; Kubzansky et al., 2004)
- But the factor analysis is precisely what we might expect given that "strongly disagree" to 4-6 does not tell us much about positive optimism



# Synthesis

1) Evidence that one factor explains the covariance structure does not indicate that the underlying causally relevant constituents are unidimensional; different indicators might well correspond to different facets of what is a multidimensional construct phenomenon having different causal effects
2) Evidence that one factor explains the covariance structure is entirely consistent with two or more “factors” that causally affect one another
3) Evidence that two factors explain the covariance structure is entirely consistent with the underlying model generating the data being univariate

Does factor analysis ever “work”…?

# Comprehensive Measure of Meaning (CMM)

Measure development for CMM draws on emerging consensus in psychology of a tripartite division of meaning into coherence, significance, and purpose (George and Park, 2016, 2017; Martela and Steger, 2016)

The CMM employs further distinctions in the philosophical literature for an even more fine-grained seven-fold assessment (Hanson and VanderWeele, 2021)

Coherence (Meaning of Life)

- Global Coherence
- Individual Coherence

Significance (Meaning in Life)

- Subjective Significance
- Objective Significance

Direction (Purpose in Life)

- Mission
- Purposes
- Goals

3 items chosen on conceptual grounds (out of a pool of 700) for each subdomain

Causal relations between domains are likely; ignoring this and proceeding with…

Exploratory factor analysis on a sample of 4,058 UBC students with 7 factors…

# Comprehensive Measure of Meaning (Padgett et al., 2023)

| item | f1 | f2 | f3 | f4 | f5 | f6 | f7 |
|---|---|---|---|---|---|---|---|
| CG1 | **0.92** | 0.00 | -0.03 | 0.00 | -0.02 | 0.01 | 0.01 |
| CG2 | **0.87** | 0.02 | 0.00 | 0.02 | 0.02 | 0.00 | -0.04 |
| CG3 | **0.43** | **0.45** | 0.00 | 0.10 | 0.00 | -0.03 | 0.01 |
| CI1 | **0.49** | 0.13 | 0.23 | -0.02 | 0.00 | 0.16 | 0.00 |
| CI2 | 0.00 | **0.50** | 0.30 | 0.09 | -0.07 | 0.01 | -0.01 |
| CI3 | 0.08 | **0.48** | 0.30 | -0.01 | 0.01 | 0.06 | 0.01 |
| SS1 | 0.01 | 0.09 | **0.73** | 0.07 | -0.01 | 0.01 | -0.02 |
| SS2 | -0.02 | 0.10 | **0.67** | 0.02 | 0.02 | -0.01 | 0.00 |
| SS3 | 0.21 | -0.02 | **0.62** | 0.00 | 0.13 | 0.00 | 0.03 |
| SO1 | -0.09 | 0.09 | -0.01 | **0.62** | 0.03 | 0.04 | 0.02 |
| SO2 | 0.03 | -0.01 | 0.05 | **0.78** | -0.01 | -0.09 | 0.03 |
| SO3 | 0.00 | 0.00 | **-0.31** | **1.07*** | 0.00 | 0.00 | -0.01 |
| DM1 | 0.26 | 0.00 | 0.02 | 0.23 | **0.37** | 0.08 | -0.07 |
| DM2 | -0.01 | 0.11 | 0.00 | 0.00 | **0.91** | 0.00 | 0.01 |
| DM3 | 0.01 | 0.00 | -0.10 | 0.01 | **0.50** | **0.52** | 0.01 |
| DP1 | 0.01 | 0.00 | 0.00 | 0.02 | 0.01 | **0.89** | -0.01 |
| DP2 | 0.21 | 0.01 | 0.06 | 0.00 | 0.09 | **0.55** | 0.03 |
| DP3 | -0.01 | 0.00 | 0.07 | 0.00 | -0.11 | 0.25 | **0.55** |
| DG1 | 0.11 | -0.13 | 0.00 | 0.04 | -0.01 | 0.06 | **0.77** |
| DG2 | -0.02 | 0.05 | -0.03 | -0.01 | 0.03 | 0.00 | **0.80** |
| DG3 | 0.04 | 0.00 | 0.02 | -0.01 | 0.02 | -0.11 | **0.81** |

# A Re-Interpretation of Factor Analysis

- Factor analysis can give insight into the covariance structure, and into which indicators are more strongly correlated with one another
- However, correlations can themselves arise because of conceptual relations, or causal relations, or because of assessing similar portions of the distribution of the underlying phenomena

Re-Interpretation: Factor analysis identifies groups of indicators with shared underlying variation that may arise from conceptual relations, causal relations, or similar distributional coverage

- Factor analysis is one approach to grouping indicators based on correlations that may themselves arise for multiple reasons
- It is a meaningful way to draw particular types of distinctions
- However, there are also other reasons, or criteria, for grouping indicators together and it may thus be reasonable to supplement factor analysis…

# Supplementing Factor Analysis: Future Directions

Distinctions between indicators might be drawn by considering whether there are individuals with extreme differences between indicators

- ❖ Quantiles of Extreme Differences (QED) Matrix (VanderWeele and Padgett, 2024)

Distinctions and similarities across indicators might be assessed based on correlations among observed individual-centered residuals for indicators (more agnostic to factor structure)

- ❖ Relative Excess Correlation (REC) Matrix and Observed Residual Correlation (ORC) Matrix (VanderWeele and Padgett, 2024)

Distinctions (and similarities) across indicators might be assessed based on differences in causal effects on outcomes across indicators… or based on differences/similarities in effects of interventions on indicators

- ❖ Counterfactual decomposition into common factor effects and residual effects (Padgett and VanderWeele, 2024)



These challenges, and the newer approaches, together suggest…

# Conclusions

- **Factor analysis** should <u>not</u> be viewed as the definitive method for assessing commonalities and distinctions across indicators
- Groupings of indicators in factor analysis can arise from conceptual relations, causal relations, or similar distributional coverage
- The assumption of a <u>univariate</u> *structural* latent is often just presumption, and can (and should) be tested
- Causal relationships make supposed factor “discovery” challenging
- Special care is needed with factor analysis when using negative items

- Most **psychosocial constructs** are inherently multi-dimensional
- We can still summarize multidimensional phenomena by univariate measures, but should be aware of what is lost when we do so
- Different facets of a phenomenon may have different casual effects
- New counterfactual decomposition of individual indicator associations into “common factor” effects vs. residual effects may hold promise
- Factor analysis can still be useful, and sometimes incredibly powerful, but requires a more careful re-interpretation

# Additional Slides

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# Multiple Versions of Treatment Theory

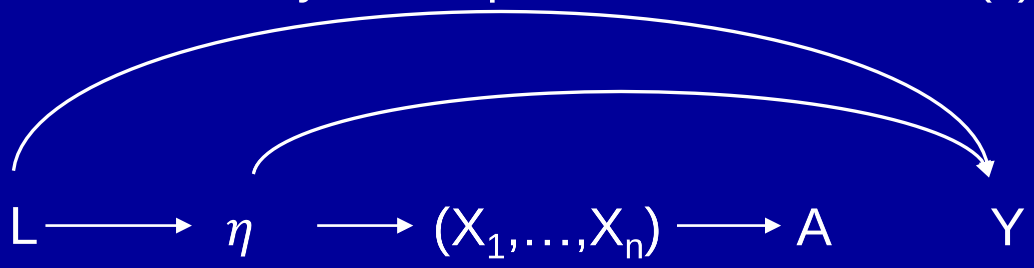
Suppose then that there is some underlying latent *multivariate* “version of treatment” variable K = $\boldsymbol{\eta}$ giving rise to indicators $(X_1,\ldots,X_n)$

Let $A = f(X_1,\ldots,X_n)$ denote some constructed measure

Let L denote some set of measured covariates

Let Y be the outcome, and Y(k) the potential outcome if K had been k

- Suppose outcome Y is independent of A conditional on L and K
- Suppose that $Y(k) \perp\!\!\!\perp K \mid L$ i.e. no confounding of the effect of K on Y conditional on covariates L
- Suppose also consistency assumption hold such that Y(k)=Y when K=k



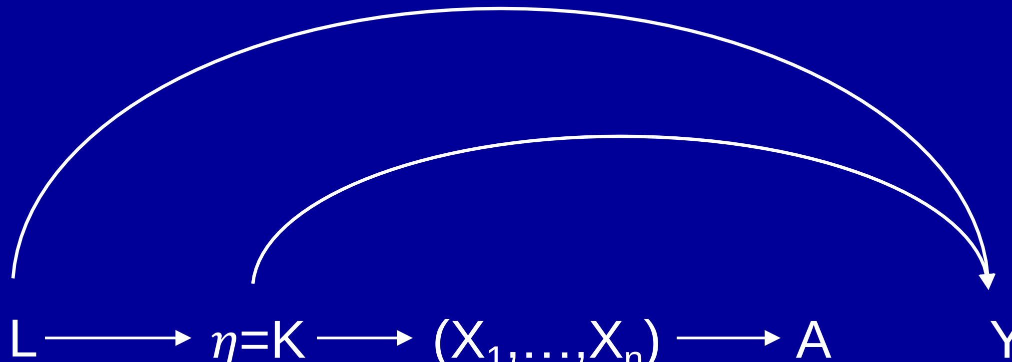

The following result is a generalization of prior work on MVT (VanderWeele and Hernan, 2013) weaking a many-to-one map from version K to treatment A to the independence assumption

# Multiple Versions of Treatment

Theorem (VanderWeele, 2022): If $Y(k) \perp\!\!\!\perp K \mid L$ and $Y \perp\!\!\!\perp A \mid (K,L)$ then

$$\sum_l \mathrm{E}(Y|A=a,l)pr(l) - \sum_l \mathrm{E}(Y|A=a^*,l)pr(l) = \sum_{l,k} \mathrm{E}\{Y(k)|l\}pr(K=k|A=a,l)pr(l) \\ - \sum_{l,k} \mathrm{E}\{Y(k)|l\}pr(K=k|A=a^*,l)pr(l).$$

This latter causal expression can itself be interpreted as a comparison in a randomized trial in which, within strata of covariates L, each arm is randomly assigned a "version of treatment" K from the observed distribution of K in the population amongst those with A=a and L=l versus A=a* and L=l



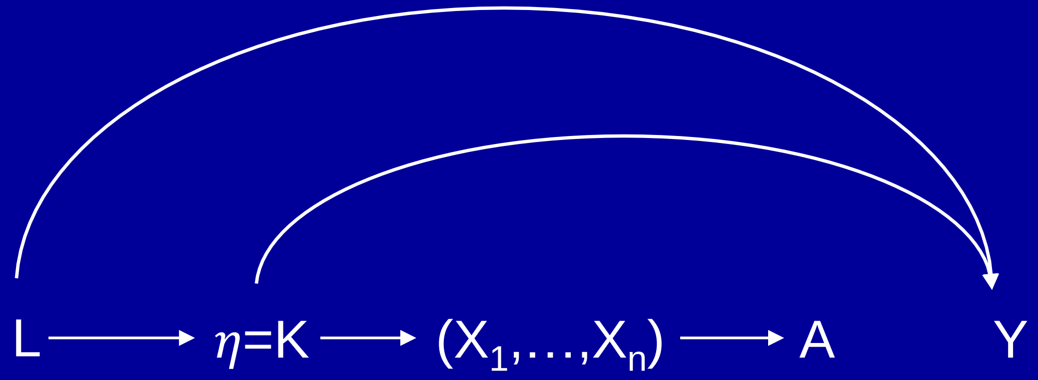
# Challenges in Interpretation

- If we do not know the “versions of treatment” or relevant underlying reality $\eta$, the interpretation is somewhat ambiguous
- We have no way of implementing the “intervention” in practice
- If we do not know all of the underlying “versions of treatment”, it is more difficult to assess the no-confounding assumption
- Interpretation of the distribution of “versions” in fact varies depending on what adjustments are made in L

- But this is arguably the best we can do with regard to a potential causal interpretation of ill-defined “treatments” or… to measures we construct from a series of indicators related to our multidimensional constructs…
- Awareness of these limitations ultimately helps with interpretation

# Interpretation of Classical Measurement Model Analyses

The interpretation is applicable with a univariate reflective model but is more general, and does not presuppose a univariate structural model




However, proceeding with a univariate measure…

(1) Can obscure what is driving the effects

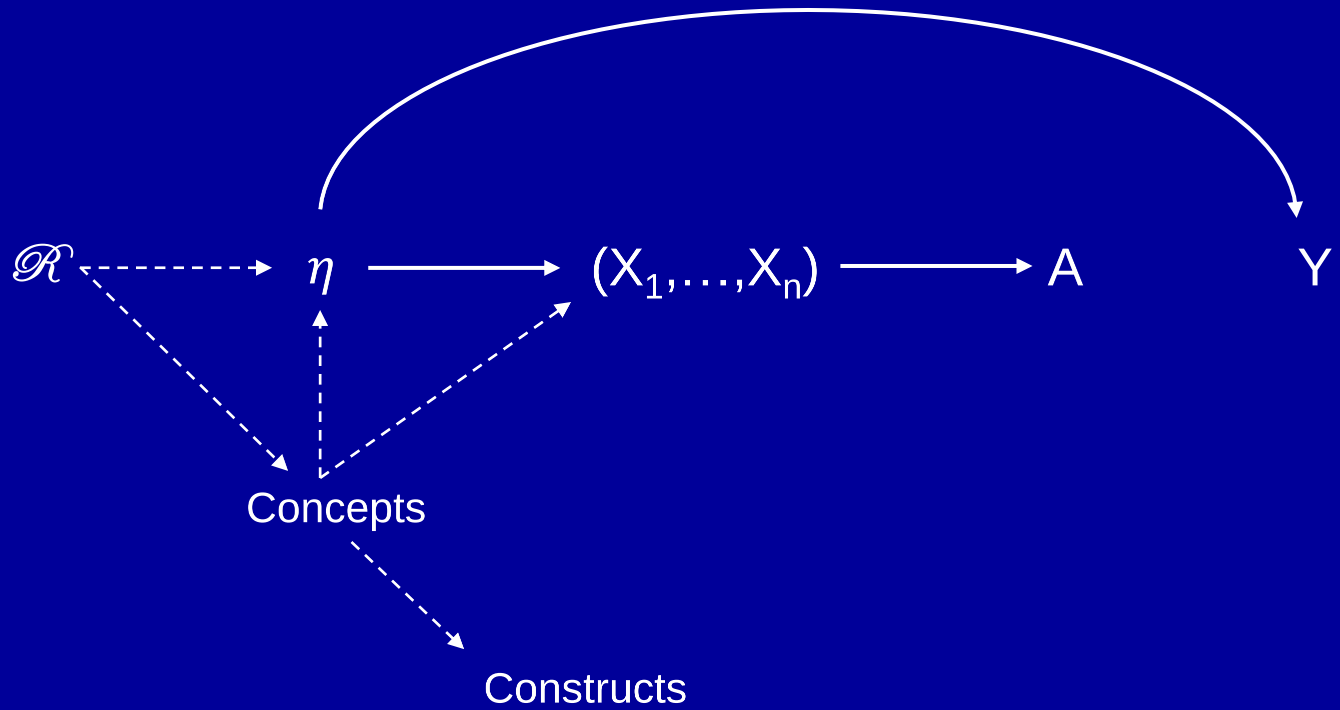
(2) If the 'causally relevant' indicators are rare, effects can be missed

(3) If the 'causally relevant' indicators are inversely correlated with the total score, the direction of effect can be wrong

(4) Does not provide guidance on where to potentially intervene

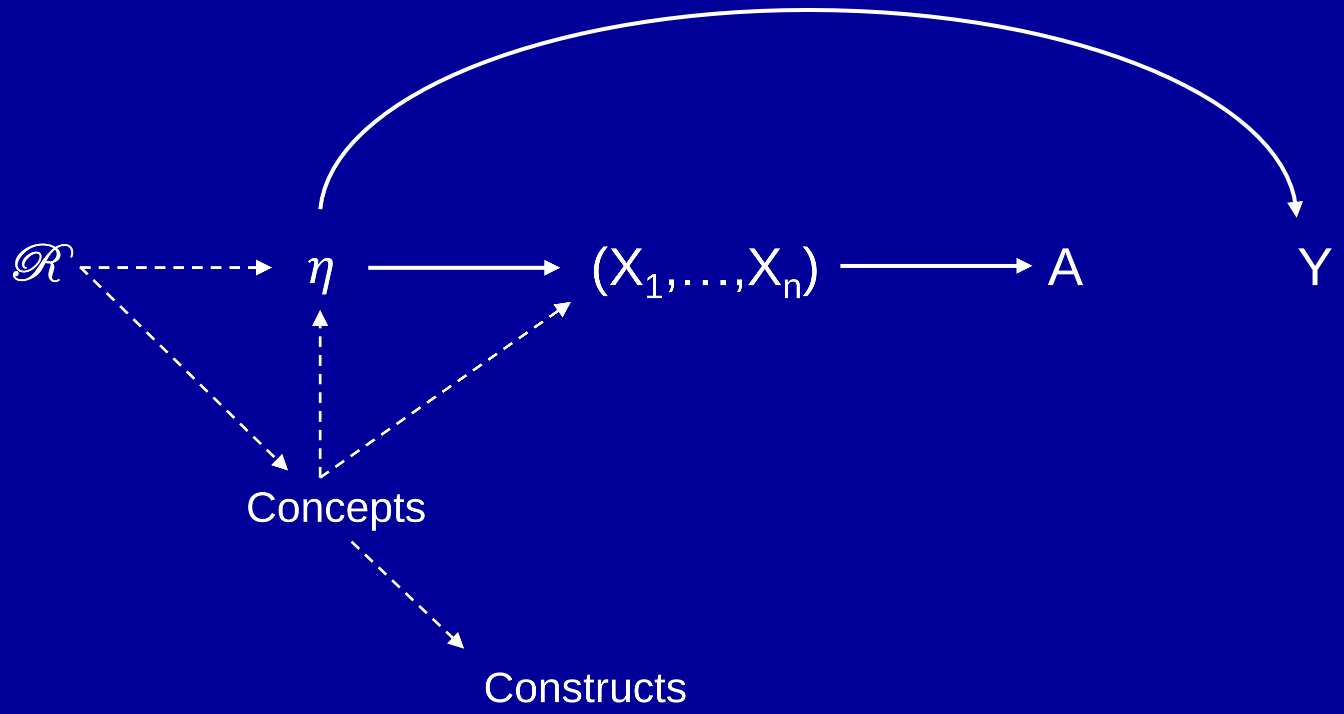
(5) It may thus be more informative to do indicator-specific analyses

# New Model for Measurement



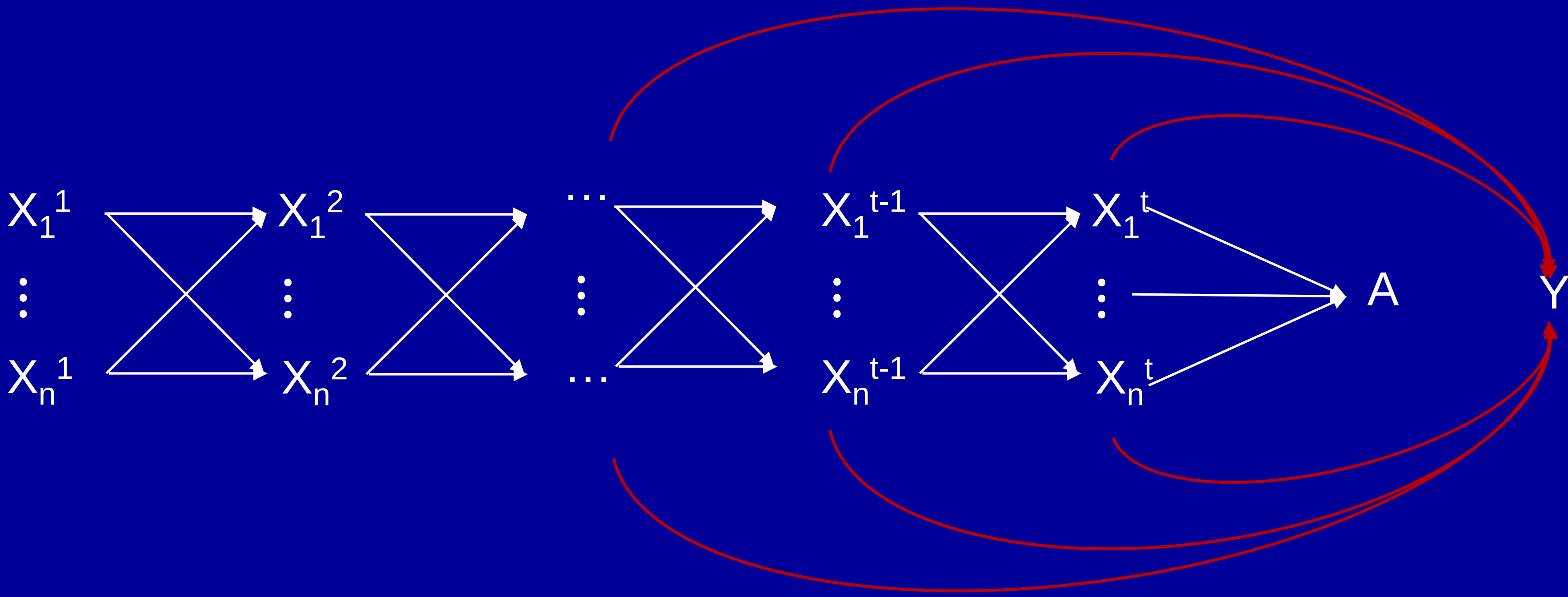

- Underlying reality $\mathcal{R}$ gives rise to our concepts from we specify constructs
- Certain features of that underlying reality (denoted by multi-dimensional variable $\boldsymbol{\eta}$) are relevant to the construct and causally related to outcome Y
- These aspects of reality also give rise to a set of observed indicators $(X_1,\ldots,X_n)$ from which we might also form a measure A (VanderWeele, 2022)

# New Model for Measurement

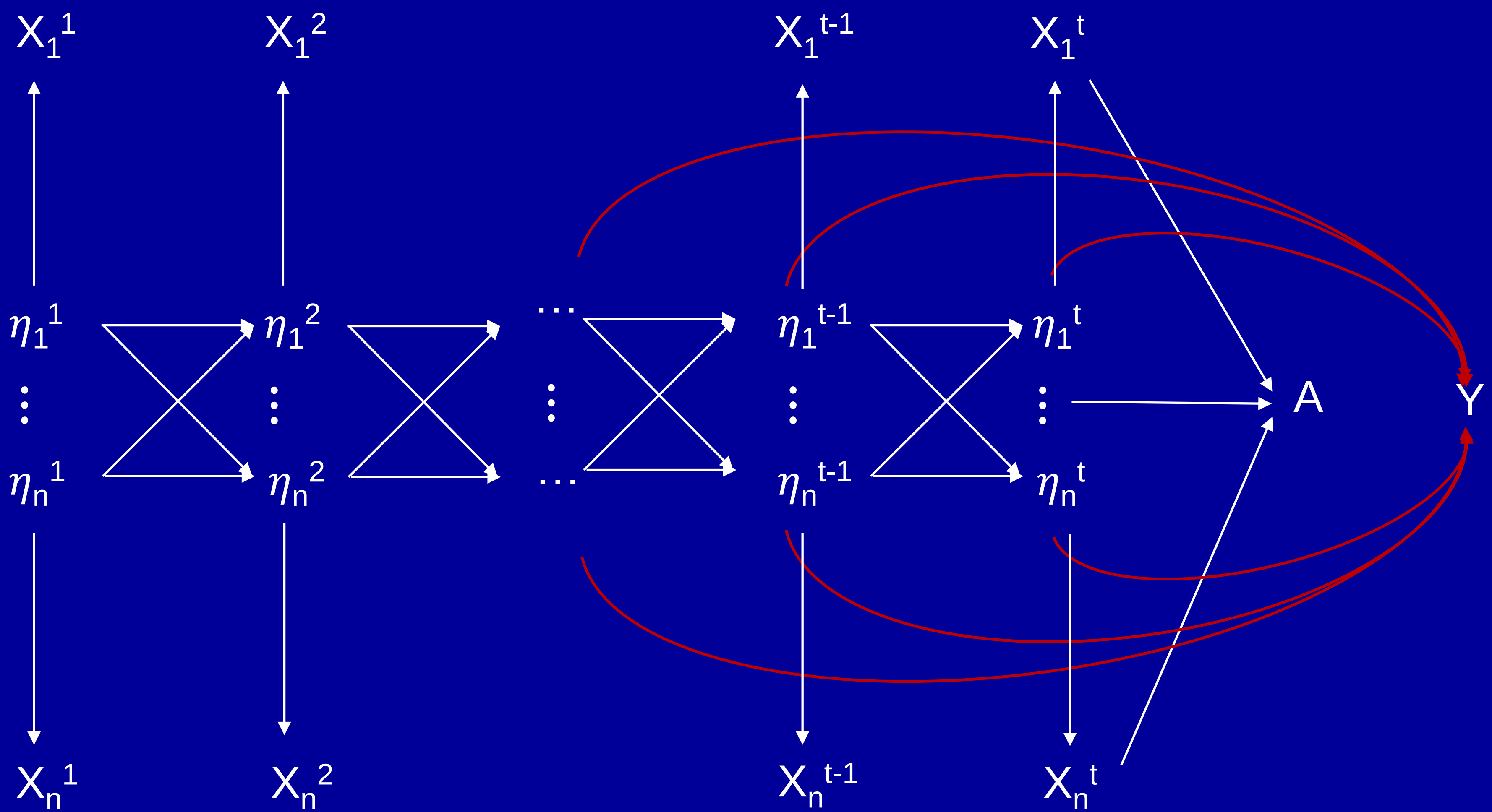



- Under MVT, there is radical freedom: we can use any A or $(X_1,\ldots,X_n)$
- But interpretation can often be more obscure
- We need a better mapping between constructs and items (analytic philosophy)
- We need a better construct definitions (analytic philosophy)
- We need to understand the phenomena giving rise to item responses $(X_1,\ldots,X_n)$ (cognitive testing)

# More Complex Models



# More Complex Models



# Example: Social Integration

Chang et al. (2017) use data from the Nurses Health Study (n=76,362) to examine potential effects of social integration on incident coronary heart disease (CHD) using the Berkman-Syme Social Integration Index in 1992, following incident CHD through 2014

Social Support Indicators: (i) Number of close friends, (ii) Community group participation, (iii) Religious service attendance, (iv) Marital status

Analyses adjusted for age, education, husband's education, census-tract income, hypertension, diabetes, cholesterol, family MI history, depressive symptoms

Compared Low vs. High Social Integration

Proportional Hazard Model: HR = 0.79 (95% CI: 0.70, 0.88)

But this might obscure distinct effects of different aspects of social integration
If we consider the analysis with each component of social integration…

| Social Integration Components | Total CHD | |
|---|---|---|
| | No of Cases | HR (95% CI)† |
| Religious attendance | | |
| Almost never | 608 | 1.00 (referent) |
| 1–3/mo to <once/mo | 388 | 0.97 (0.85–1.10) |
| Once/wk | 955 | 0.97 (0.87–1.07) |
| >once/wk | 398 | 0.82 (0.72–0.93) |
| Community group participation per week | | |
| 0 h | 858 | 1.00 (referent) |
| 1–2 h | 625 | 0.95 (0.86–1.05) |
| 3–10 h | 718 | 1.00 (0.90–1.11) |
| 11+ h | 121 | 1.10 (0.90–1.34) |
| Number of close friends | | |
| 0 | 52 | 1.00 (referent) |
| 1–2 | 370 | 0.94 (0.70–1.25) |
| 3–9 | 1493 | 1.02 (0.77–1.35) |
| 10+ | 410 | 1.02 (0.76–1.37) |
| Marital status | | |
| Unmarried or unpartnered | 765 | 1.00 (referent) |
| Married or partnered | 1597 | 0.94 (0.85–1.03) |

# CHD

Associations seems most pronounced with religious service attendance

Similar patterns emerge with fatal CHD where marital status also emerges as perhaps exerting moderate effects (Chang et al., 2017)

Authors report similar results when components are mutually adjusted for one another

| | | HR and 95%CI | *P* Value | | HR and 95%CI | *P* Value |
|---|---|---|---|---|---|---|
| Religious service attendance in 1996 [a] | Once or more /week, yes vs.no | 0.17 (0.07-0.47) | .0005 | Once or more /week, yes vs.no | 0.16 (0.06-0.43) | .0003 |
| Social integration score in 1996 with service attendance excluded [b] | Continuous, per 1SD increase [e] | 0.82 (0.52-1.28) | .38 | ≥ 7 vs. < 7 [f] | 1.28 (0.59-2.75) | .53 |
| Social integration score included religious service attendance in 1996. (the original definition version of social integration score) [c] | Continuous, per 1SD increase [e] | 0.67 (0.41-1.10) | .11 | ≥ 6 vs. < 6 [f] | 0.83 (0.39-1.78) | .63 |
| Components of social integration score in 1996 [d] | Currently married (yes, vs. no) | 1.60 (0.52-4.92) | .41 | yes vs. no | 1.46 (0.47-4.57) | .52 |
| | Number of close friends (continuous) [e] | 1.17 (0.75-1.81) | .48 | ≥ 3 vs. < 3 [f] | 0.84 (0.27-2.66) | .77 |
| | Number of close relatives (continuous) [e] | 0.84 (0.56-1.24) | .38 | ≥ 3 vs. < 3 [f] | 1.37 (0.55-3.38) | .50 |
| | Close relatives seen at least once/month (ordinal) [e] | 0.99 (0.65-1.50) | .95 | ≥ 2 vs. < 2 [f] | 0.66 (0.28-1.59) | .36 |
| | Close friends seen at least once/month (ordinal) [e] | 0.64 (0.38-1.08) | .10 | ≥ 3 vs. < 3 [f] | 0.78 (0.28-2.16) | .63 |
| | Hours of social group participation (continuous) [e] | 1.11 (0.80-1.53) | .53 | ≥ 2 vs. < 2 [f] | 1.22 (0.55-2.69) | .62 |

# Suicide

Similar patterns are present with suicide in NHS data (n=89,708) (VanderWeele et al., 2016; cf. Tsai et al., 2014, 2015)

For mutually adjusted components measured continuously (first columns) or dichotomously (latter columns) religious service attendance had the strongest associations

# Mortality

- Similar patterns also with all-cause mortality in NHS data (n=74,534) when components are mutually adjusted for each other (Li et al., 2016)
- Religious service attendance and marriage seem to drive the protective mortality association of social support

| | | HR and 95%CI | P Value |
|---|---|---|---|
| Religious service attendance in 1996 [a] | Once or more /week,<br><br>yes vs.no | 0.86 (0.83-0.90) | <0.0001 |
| Social integration score in 1996 [b] | Continuous, per 1SD increase | 0.97 (0.95-0.99) | 0.03 |
| Social integration score included religious service attendance in 1996. (the original definition version of social integration score) [c] | Continuous, per 1SD increase | 0.96 (0.94-0.98) | 0.0002 |
| Components of social integration score in 1996 [d] | | | |
| | Currently married (yes, vs. no) | 0.86 (0.82-0.91) | <0.0001 |
| | Number of close friends (continuous) | 1.03 (1.01-1.05) | 0.007 |
| | Number of close relatives (continuous) | 0.99 (0.97-1.01) | 0.27 |
| | Seen close relatives at least once/month (yes vs. no) | 0.99 (0.97-1.01) | 0.51 |
| | Seen close friends at least once/month (yes vs. no) | 1.01 (0.99-1.04) | 0.27 |
| | Hours of social group participation (continuous) | 1.01 (0.99-1.03) | 0.25 |